ERRATA FOR ‘FUNDAMENTALS OF PURE MATHEMATICS’ LECTURE NOTES

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p.19, l.1 Remove ‘()’.

p.53, l.13 Replace ‘min’ by ‘inf’ or ‘glb’.

p.58 The proof of Theorem 19.5 is as follows:

Proof of 19.5. Let A be infinite. Define a map $f : \mathbb{N} \to A$ inductively as follows:

- Choose any $a \in A$ and define $f(1) = a$.
- Suppose $f(1), \ldots, f(n)$ have been defined.
  Choose $b \in A - \{f(1), \ldots, f(n)\}$ and define $f(n+1) = b$. (We can do this because A is infinite.

The map f is injective. So $|\mathbb{N}| < |A|$. $\boxed{19.5}$

p.59, l.21 Replace with

$$x \mapsto \pi(x - \frac{1}{2})$$

p.62 In the statement of Theorem 21.2, delete the word ‘nonempty’.

p.64 Theorem 23.3 and its proof are *not* examinable.

p.65 The given proof of Theorem 24.2 is incorrect. Correct version is as follows:

Proof of 24.2. Define $f : (0,1) \to \mathbb{P}(\mathbb{N})$ by

$$0.a_1 a_2 a_3 \cdots \mapsto \{i : a_i = 1\},$$

where $a_j \in \{0, 1\}$. (So $0.a_1 a_2 a_3 \cdots$ is a binary expansion.)

The mapping f is an injection, so $|(0,1)| \leqslant |\mathbb{P}(\mathbb{N})|$. The mapping f is not, however, a surjection: there is no $x \in (0,1)$ with $f(x) = \mathbb{N} \in \mathbb{P}(\mathbb{N})$. ($0.1111 \cdots$ does not work: remember that for *decimal* expansions, $0.999 \cdots = 1.000 \cdots$; similarly, for *binary* expansions, $0.111 \cdots = 1.000 \cdots \notin (0,1)$.)

Define $g : \mathbb{P}(\mathbb{N}) \to (0,1)$ by

$$g(X) = 0.0a_1 0a_2 0a_3 \cdots \quad (a_i = 1 \iff i \in X).$$

(The more obvious definition of $g(X) = 0.a_1 a_2 a_3 \cdots$ doesn’t work since it gives $g(\mathbb{N}) = 0.111 \cdots = 1 \notin (0,1)$ as discussed above.)

This mapping g is easily seen to be an injection.

So, by the Schröder–Bernstein Theorem, $|\mathbb{R}| = |(0,1)| = |\mathbb{P}(\mathbb{N})|$. $\boxed{24.2}$