

Group Recognition

Max Neunhoffer

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# Group Recognition

Max Neunhoffer



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GAC 2010, Allahabad

# Constructive recognition

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Let  $\mathbb{G}$  be some ambient group and

$$M_1, \dots, M_k \in \mathbb{G}.$$

Find for  $G := \langle M_1, \dots, M_k \rangle$ :

- The group order  $|G|$  and
- an algorithm that, given  $M \in G$ ,
  - **decides**, whether or not  $M \in G$ , and,
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If this problem is solved, we call

$\langle M_1, \dots, M_k \rangle$  recognised constructively.

# GAP examples

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# What is a reduction?

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- e.g.  $H \leq \Sigma_m$  or  $H \leq \text{GL}_n(\mathbb{F}_q)$  with **smaller**  $m$  or  $n, q$  respectively.

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→ Monte Carlo algorithm to compute  $N$

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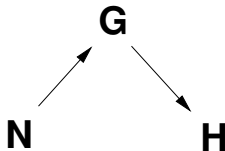
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# Recursion: composition trees

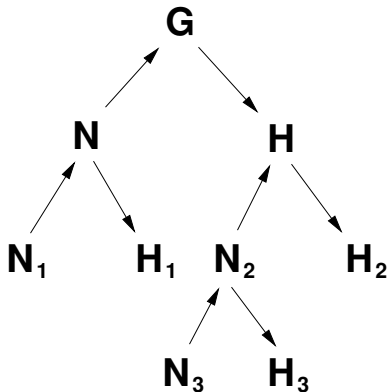
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Up arrows: inclusions  
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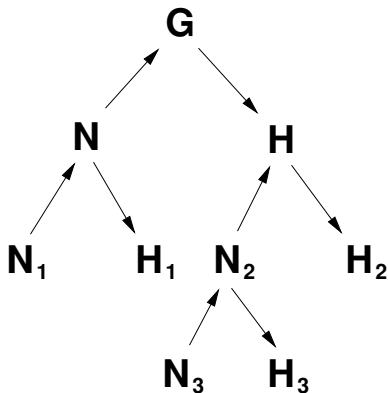


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Old idea, improvements are still being made.

## Example: invariant subspace

Let  $V = \mathbb{F}_q^{1 \times d}$  and  $G \leq \text{GL}_d(\mathbb{F}_q)$ , then  $G$  acts on  $V$ .

Let  $W \leq V$  be an **invariant subspace**, i.e.:

$$WM = W \quad \text{for all } M \in G$$

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Choose basis  $(w_1, \dots, w_e)$  of  $W$  and extend to a basis

$$(w_1, \dots, w_e, w_{e+1}, \dots, w_d)$$

of  $V$ . After a **base change** the matrices in  $G$  look like this:

$$\left[ \begin{array}{c|c} A & \mathbf{0} \\ \hline C & D \end{array} \right] \quad \text{with } A \in \mathbb{F}_q^{e \times e}, C \in \mathbb{F}_q^{(d-e) \times e}, D \in \mathbb{F}_q^{(d-e) \times (d-e)}$$

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Together with a reduction additional information is gained!

# Long SLPs

Typical examples:

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Typical elements in  $W$  give SLPs of length  $\approx 500$ .

# Comparison

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We compare lengths of SLPs:

	Stabiliser chain in strong		Composition tree in gens
$G$	15		900
$\Sigma_{12} \wr \Sigma_5$	500		10000

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We compare lengths of SLPs:

	Stabiliser chain		Composition tree	
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$G$	15	290		900
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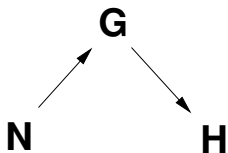
We compare lengths of SLPs:

	Stabiliser chain		Composition tree	
	in strong	in gens	in nice	in gens
$G$	15	290	15	900
$\Sigma_{12} \wr \Sigma_5$	500	4300	300	10000

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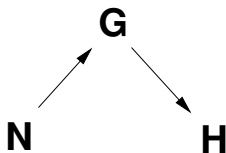
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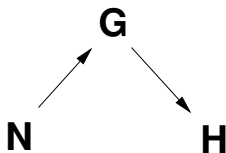


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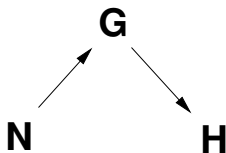
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**Note:** The first allows to compute  $N$  once  $H$  is recognised!

# Constructive recognition revisited

## Problem — new formulation

Let  $\mathbb{G}$  be  $\Sigma_n$  or  $\mathrm{GL}_n(\mathbb{F}_q)$  or  $\mathrm{PGL}_n(\mathbb{F}_q)$  and

$$M_1, \dots, M_k \in \mathbb{G}.$$

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If this problem is solved, we call

$\langle M_1, \dots, M_k \rangle$  **recognised constructively.**

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- and, given preimages of the original generators of  $G$  under some homomorphism, we can find preimages of the nice generators.

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This approach implements the **asymptotically best known algorithms** for permutation groups **in the composition tree framework**.

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## Theorem (Aschbacher, 1984)

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The classes C1 to C7 are defined “geometrically” and *promise some reduction*.

The classes C8 and C9 have to be dealt with as *leaves of the composition tree*.

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