

UNIVERSITY OF ST ANDREWS
MT5826 Finite Fields
Tutorial Sheet: Chapter 5

1. Using any method you choose, find a primitive element of \mathbb{F}_9 . Demonstrate, by direct verification, that each of its conjugates with respect to \mathbb{F}_3 is also a primitive element.
2. The set of automorphisms of \mathbb{C} over \mathbb{R} (i.e. the automorphisms of \mathbb{C} which fix \mathbb{R} pointwise) forms a group. Describe this group.
Hint: begin by considering the effect of such an automorphism on $i \in \mathbb{C}$.
3. Let $K = \mathbb{F}_q$ and let F be a finite extension of K . Let $\alpha = \beta^q - \beta$ for some $\beta \in F$.

Prove that

$$\alpha = \gamma^q - \gamma \text{ with } \gamma \in F \Leftrightarrow \beta - \gamma \in K.$$

4. Let $K = \mathbb{F}_q$ and let $F = \mathbb{F}_{q^m}$ be a finite extension of K .
Prove that: for $\alpha \in F$,

$$N_{F/K}(\alpha) = 1 \Leftrightarrow \alpha = \beta^{q-1} \text{ for some } \beta \in F^*.$$

5. Prove that, if the order of basis elements is taken into account, then the number of different bases of \mathbb{F}_{q^m} over \mathbb{F}_q is

$$(q^m - 1)(q^m - q)(q^m - q^2) \cdots (q^m - q^{m-1}).$$