UNIVERSITY OF ST ANDREWS
School of Mathematics and Statistics
MT4603 Groups: Tutorial 1.

1. Write the following permutations from S_7 as products of disjoint cycles:

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 2 & 5 & 6 & 4 & 7 \end{pmatrix}, \ \beta = (2\ 3\ 4)(1\ 2\ 5)(3\ 6\ 1\ 7).$$

2. Consider the symmetric group S_n.

(i) Prove that $(i_1\ i_2\ \ldots\ i_m)^{-1} = (i_m\ i_{m-1}\ \ldots\ i_1)$.

(ii) Prove that disjoint cycles commute.

(iii) Let $\sigma \in S_n$ be any permutation, and let $\sigma = \gamma_1\gamma_2\ldots\gamma_m$ be its decomposition into a product of disjoint cycles. Prove that for every integer i we have $\sigma^i = \gamma_1^i\gamma_2^i\ldots\gamma_m^i$.

3. Let G be a group and let $a \in G$ be arbitrary. Define a mapping $f_a : G \longrightarrow G$ by $xf_a = a^{-1}xa$. (This mapping is called *conjugation by a*.)

(i) Prove that f_a is an isomorphism.

(ii) Find $a \in G$ such that f_a is the identity mapping (i.e. $xf = x$ for all $x \in G$).

(iii) Prove that G is abelian if and only if all the mappings f_a are equal to the identity mapping.

4. Let G be a group, and define a mapping $f : G \longrightarrow G$ by $xf = x^{-1}$. Prove that f is a bijection. Also, prove that f is an isomorphism if and only if G is abelian.

5. Let G be a group, and let $a, b \in G$. Prove that if $a^2 = e$ and $b^2a = ab^3$ then $b^5 = e$. (Hint: $b^4 = b^2b^2aa$, $b^6 = b^2b^2b^2$.)

6. Prove that a group G is abelian if and only if the equality $(ab)^i = a^ib^i$ holds for all $a, b \in G$ and for three consecutive integer values of i. Give an example which shows that the above statement is no longer valid if ‘three’ is replaced by ‘two’.