UNIVERSITY OF ST ANDREWS
School of Mathematics and Statistics
MT4603 Groups: Tutorial 3.

1. Consider the dihedral group D_4, and let H be the subgroup generated by $(1\ 2)(3\ 4)$. List the left and right cosets of H in D_4.

2. Let G be a group, and let $H, K \leq G$. Denote by C_H, C_K and $C_{H\cap K}$ the sets of all right cosets in G of H, K and $H \cap K$ respectively. Define a mapping $\phi : C_{H\cap K} \longrightarrow C_H \times C_K$ by $((H \cap K)a)\phi = (Ha, Ka)$. Prove that ϕ is well defined and that it is one-one. Conclude that $[G : H\cap K] \leq [G : H][G : K]$, and that the intersection of two subgroups of finite index also has finite index.

3. Prove that the symmetric group S_4 contains a subgroup of order m for every m dividing the order of S_4.

4. (i) Prove that if a group G of order 4 is not cyclic, then every non-identity element of G has order 2.

(ii) Prove that up to isomorphism $\mathbb{Z}_4$ and K_4 are the only groups of order 4.

(iii) Describe all the groups of order less then six; are they all abelian?

(iv) Does there exist a non-abelian group of order 6? Order 7? Order 8?

5. Let $f : G \longrightarrow H$ be a homomorphism. Prove that f is a monomorphism (i.e. an injective homomorphism) if and only if $\ker f = \{e\}$.

6. We say that a group H is a homomorphic image of a group G if there exists an epimorphism (i.e. an onto homomorphism) $G \longrightarrow H$. Prove that every homomorphic image of a cyclic (respectively, abelian) group is cyclic (respectively, abelian).