

UNIVERSITY OF ST ANDREWS
School of Mathematics and Statistics
MT4603 Groups: Tutorial 8.

1. Prove that S_n is soluble for $n = 1, 2, 3, 4$ and is not soluble for $n \geq 5$.
2. If H is a subgroup of a group G prove that $H^{(n)} \subseteq G^{(n)}$ for all $n \geq 0$. Conclude that every subgroup of a soluble group is also soluble.
3. Prove that the direct product $G \times H$ is soluble if and only if both G and H are soluble. (Hint: Use $(G \times H)' = G' \times H'$.)
4. List the conjugacy classes of the symmetric group S_5 , by giving one element from each conjugacy class and size of each conjugacy class. Are the elements $(1\ 2\ 3\ 4\ 5)$ and $(1\ 2\ 3\ 5\ 4)$ conjugate in the alternating group A_5 ?
5. List all the normal subgroups of the symmetric group S_4 . (Hint: A subgroup is normal if and only if it is a union of conjugacy classes.)
6. Let G be a group, let $a \in G$, let τ_a be the inner automorphism $x\tau_a = a^{-1}xa$, and let ϕ be any automorphism of G . Prove that $\phi^{-1}\tau_a\phi = \tau_{a\phi}$. Prove that $\text{Inn}(G) \trianglelefteq \text{Aut}(G)$.